The Y of Wavicles — Answers

U. Nandivada and J. D. D. Martin
Department of Physics and Astronomy, University of Waterloo

Questions for students related to the video: Wavicles and the weakest bond — can two helium atoms form a molecule? discussing the paper: `https://doi.org/10.1063/1.470772`

1 Introductory (high-school/first year university)

See Ontario High-School Curriculum References.

(1) **Question:**
What have you learned so far about elements like helium that make it hard to believe a bond is likely to form between two of their atoms?

Answer:
Helium is a noble gas. Typically these atoms do not react with other atoms. Based on what we learned so far, we understand that this is because Helium has a full valence shell, like other noble gases. Since it has a full shell, it won't tend to bond with other atoms to make a full shell.

(2) **Question:**
The speed of light in a medium can be related to the light's frequency and is modeled using the equation $v = f\lambda$. In this equation, v is the light's speed in that medium, f is the light wave's frequency, and λ is the light's wavelength. Using this equation, convince yourself why "frequency doubling" is the same as saying "wavelength halving" (although one rolls off the tongue a bit better). (Hint: If you would like to use numbers, pick any frequency you want and double it! See what happens to the wavelength.)

Answer:
The equation presented, $v = f\lambda$ relates the frequency and the wavelength of light. Let's rearrange this to isolate for λ:

$$\lambda = \frac{v}{f}.$$

Now, let's imagine that we have one light beam travelling at frequency f, like 300 THz. (A

Terahertz is 1 trillion Hertz or 10×10^{12} Hz). If the frequency doubles, then it goes to 600 THz. At 300 THz, the wavelength of light would end up being about 1 μm, or 1 micrometer. A micrometer is a millionth of a meter, or 1×10^{-6} m. At the doubled frequency, the light would be twice the wavelength, or 0.5 μm. We can also express this algebraically (that is, in variable form)

Let's go back to the equation above. If we replace f with $2f$, then you obtain:

$$\lambda = \frac{v}{2f}.$$

If you adjust the equation to separate the variables v and f from the constant number, you get:

$$\lambda = \frac{1}{2}\frac{v}{f},$$

which can be used to present the doubled wavelength in terms of the original wavelength as,

$$\lambda_{doubled} = \frac{1}{2}\lambda_{original}.$$

As shown, the wavelength corresponding to frequency doubled light is at half the value of its original.

(3) **Question:**
If helium atoms are travelling with a wavelength of 1 micrometer and the angle between the first order constructive diffraction spots and the central maximum is 5° , then how far apart are the lines in the gratings (this is the grating spacing)?

Using the de Broglie Relationship, that is, $h = \frac{\lambda}{mv} = \frac{\lambda}{p}$, determine the momentum of the helium atoms.

Answer:
We know that the equation for constructive interference from light passing through a diffraction grating takes the form:

$$d \sin(\theta) = m\lambda.$$

Where d is the grating spacing, is the angle of the light's deviation from the incident direction of travel, m is the order of diffraction, and λ is the wavelength of the light in nanometers.

We are given the wavelength and the angle of the first order constructive interference diffraction spot, so we have enough information to evaluate the equation to solve for the grating spacing.

Doing so gives a spacing of $d = 1.14 \times 10 \times 10^{-4}$ m or about 0.114 mm. This length is about as small as the human eye can see without a microscope!

The momentum of the Helium atoms can be calculated using h = 6.626×10^{-34} Js. When we rearrange the deBroglie matter-wave equation to isolate for the momentum, p, we obtain:

$$p = \frac{\lambda}{h},$$

Giving $p = 1.5 \times 10^{27}\,\frac{\text{kgm}}{\text{s}}$.

(4) **Question:**
Consider what would happen if two atoms were too close together. Why would they not want to bond? Is that the same reason for why two atoms too far apart would not want to bond? Explain your reasoning.

Answer:
In the case where the atoms are too close together, they are like particles and will repel each other. In the case where they are too far apart, the magnitude of the attractive force is far too weak to have a non-negligible impact, so they won't repel each other, but they are too far apart to interact at all.

(5) **Question:**
If red light at 632.8 nm gets frequency doubled through a crystal then diffracted through a grating, some of the red light produces a pattern along side the frequency doubled light. If both the red and its frequency doubled light enter through the same grating, and the blue 3^{rd} order constructive interference maxima are at an angle 45° from the incident direction, then what is the grating spacing?

Answer:
We know that the equation for constructive interference from light passing through a diffraction grating takes the form:

$$\sin(\theta) = m\lambda$$

Where d is the grating spacing, is the angle of the light's deviation from the incident direction of travel, m is the order of diffraction, and λ is the wavelength of the light in nanometers.

What do you know so far about the red light?

a) It has a wavelength of 632.8 nm.

b) Some of the red light gets frequency doubled.

We know the wavelength of the red light, but not anything else. We have to know some parameters about the blue light now to finish solving our problem.

What do we know about the blue light?

a) We have the angle of the light forming the 3^{rd} order maxima from the original incident direction.

b) The blue light is frequency doubled from the red, and thus its parameters relate to the red somehow.

To find the grating spacing, we can use our equation of constructive interference and evaluate it for values related to blue light.

$$d \sin \theta_{blue} = m_{blue} \lambda_{blue}$$

Since the blue light has twice the frequency of the blue light, it also has half the wavelength. This means that we can use $\frac{\lambda_{red}}{2}$ in place of λ_{blue}.

The equation becomes

$$d \sin \theta_{blue} = m_{blue} \frac{\lambda_{red}}{2}$$

All the variables are known! Now we just have to evaluate the equation, and doing so gives: $1.34 \times 10^{-6} m$ or about $1.34 \mu m$! The gratings are as far apart as the length of a bacterium!

2 Intermediate

(1) **Question:** Explain how the Heisenberg uncertainty principle prevents the two helium atoms from sitting motionless, at the bottom of the weakly attractive well describing their interaction.

Answer: The Heisenberg uncertainty relationship is:

$$m \Delta x \Delta v \geq \hbar/2$$

where m is the mass of a particle, Δx is the uncertainty in position, and Δv is the uncertainty in speed.

Although this equation is normally applied to a single particle, it can also be applied to the relative motion of two particles, where m is now the *reduced mass* of the two particle system, Δx is the separation of the two particles, and Δv is their relative speed.

If the atoms were sitting motionless, at the bottom of the attractive well, then Δx would be zero and it would be impossible to satisfy the Heisenberg uncertainty relationship.

More generally, as Δx *decreases*, then the uncertainty in Δv must *increase*.

This mathematical result tells us that the more certain we are about the separation of the two particles, the less we know about their motion relative to one another.

(2) **Question:** The peaks shown in Fig. 2 of the paper have a “width”; i.e., the signal attributed to helium dimer doesn’t just appear at one angle but over a range of angles. List as many factors as possible that could contribute to these widths.

Answer:

(1) atoms have a range of speeds, thus de Broglie wavelengths, thus diffraction angles.

(2) the detector has a certain ”width” over which it detects, collecting signal over a range of diffraction angles.

(3) the transverse coherence width of the beam will contribute to the width (smaller coherence widths lead to larger spreads).

A Ontario High-School Curriculum References

These are the curricular references made in the formation of the high school questions. Please note that these are just suggestions based on our own understanding of the Ontario High School Science Curriculum for Grades 11 and 12.

(1) Knowledge - Ontario Grade 11 Science Curriculum (SCH3U) rev. 2008: A1.9, B2.2

(2) Knowledge - Ontario Grade 12 Science Curriculum (SPH4U) rev. 2008: A1.8, A1.10, A1.12

(3) Knowledge - Ontario Grade 12 Science Curriculum (SPH4U) rev. 2008: E1.1, E2.1, E2.2, E2.3, E3.2, F2.1, F2.2

(4) Knowledge/Communication - Ontario Grade 12 Science Curriculum (SPH4U) rev. 2008: D2.1, D3.1, D3.2

(5) Application - Ontario Grade 12 Science Curriculum (SPH4U) rev. 2008: E1.1, E2.1, E2.2, E2.3, E3.2, F1.2, F3.1, F3.2